Al Khawarizmi, Vol. 9, No. 1, Juni 2025 (1 - 11)

Al Khawarizmi
Jurnal Pendidikan dan Pembelajaran Matematika

journal homepage: https://jurnal.ar-raniry.ac.id/index.php/alkhawarizmi

LEARNING DESIGN ON CUBE AND CUBOID TOPICS BASED ON LOCAL INSTRUCTION THEORY (LIT)

Tri Putri Utami
Bachelor's Program in Physical Education, Universitas Islam Kebangsaan Indonesia
Correspondence Author: triputriutami19@gmail.com

Artikel info

Artikel history:
Received 28 Juni 2025
Received in revised form 29 Juni 2025
Accepted 02 July 2025
Available online 28 July 2025

Keywords:
Local Instruction Theory (LIT); Cuboid; Cube.

Abstrak

The purpose of this study is to develop an instructional design for teaching cubes and cuboids based on Local Instruction Theory (LIT). This study employed a design research approach, which involved three phases: preparing for the experiment, designing the experiment, and conducting a retrospective analysis. The participants consisted of 32 students from Class VIII-3 at SMP Negeri 1 Banda Aceh and one teacher. In the preliminary stage, the Hypothetical Learning Trajectory (HLT) was designed based on the learning objectives, activities, and relevant media. The instructional experiment was carried out to implement the HLT, whereas the retrospective analysis was intended to evaluate the effectiveness of the applied learning trajectory. The findings indicate that a well-designed Hypothetical Learning Trajectory (HLT) can significantly improve students' understanding of cubes and cuboids.

INTRODUCTION

Mathematics is not merely about performing calculations; it also engages students in actively constructing knowledge through cognitive processes and learning experiences, aimed at enhancing their problem-solving skills in both academic contexts and everyday life. This is in line with Gee (2019), who asserts that mathematics is a discipline with the potential to foster critical thinking and reasoning abilities, as well as to equip individuals to face real-world challenges.

Geometry, particularly the study of cubes and cuboids, is one of the branches of mathematics. In classroom practice, instruction on cubes and cuboids often involves the teacher delivering information directly to students. For instance, during lessons on cube and cuboid nets, teachers frequently illustrate them on the board or refer to images in textbooks, without encouraging students to explore and construct the nets independently. Consequently, students are often unable to identify alternative cube and cuboid nets beyond those introduced by the teacher (Rahmi, 2017).

Observations conducted at SMP 1 Banda Aceh revealed that in teaching topics related to cubes and cuboids, most teachers emphasize the delivery of factual information, such as the number of edges and faces, calculations of surface area and volume, and other procedural content. Rarely do teachers encourage students to engage in higher-order thinking, such as discovering patterns when shapes are flipped or rotated, or solving for surface area and volume of irregularly composed figures. As a result, students often rely on memorization of geometric concepts and diagrams without developing a deep understanding of the inherent properties of the shapes. This leads to a lack of meaningful learning experiences and difficulties in applying mathematical concepts to real-world contexts. To overcome this issue, it is essential to develop an instructional design that effectively stimulates student interest in mathematics learning.

Teachers play a crucial role in preparing appropriate instructional designs for the teaching and learning process. Instructional design is a plan related to student activities (Dalziel, 2015). According to Confrey, Gianopulos, McGowan, & Shah (2017), a learning trajectory illustrates various ideas that tend to emerge during student-centered learning and a sequence of tasks that effectively foster understanding and support cognitive development. Therefore, before beginning the learning process, teachers need to prepare all elements related to instruction. Effective learning must be supported by well-prepared instructional designs. A teacher's ability to design learning activities that support discovery-based learning has a significant impact on students' success in understanding a concept (Dalziel, 2015). Teachers are expected to provide activities that support the discovery process so that students are able to construct their own knowledge (Yanti & Fauzan, 2021). Through this study, a product called the Local Instructional Theory (LIT) will be developed. LIT is a theory of the learning process for a specific topic that includes supporting activities (Gravemeijer, 2004).

METHODS

This study employed a design research methodology, focusing on the development of learning theory and activities through the construction of a Local Instruction Theory (LIT), in collaboration with researchers and teachers (Muhsi, Asmika & Panggabean, 2025; Mashitoh et al., 2025; Fitri & Prahmana, 2020). The research was conducted at SMP Negeri 1 Banda Aceh. The participants in this study were eighth-grade students from the same school. The first trial of the Hypothetical Learning Trajectory (HLT 1) involved six students from Class VIII-8, selected based on their academic abilities: two high-achieving, two average, and two low-achieving students. This classification was determined based on information provided by the mathematics teacher. The second cycle (HLT 2) involved 32 students from Class VIII-3 and one mathematics teacher, who also served as the model teacher for the class.

This study was conducted following the stages proposed by Gravemeijer & Cobb (2006), namely the preparation phase (preparing for the experiment), the experimental phase (design experiment), and the retrospective analysis phase. In the preparation or preliminary design phase, the Hypothetical Learning Trajectory (HLT) and instructional materials were designed. To develop the HLT, the researcher conducted a literature review, classroom observations, and took field notes. This phase aimed to observe students' responses to the designed learning activities. The data collected at this stage were analyzed as input for revising and improving the HLT for the next phase, the design experiment phase. The revised HLT was then implemented during this experimental stage. Data collection techniques included documentation, student worksheets (LKPD), interviews, and field notes. The collected data were analyzed during the retrospective analysis phase. The analysis process involved comparing the hypothesized learning trajectory with the actual learning process observed in the classroom. The general purpose of the retrospective analysis is to develop a Local Instructional Theory (LIT). Finally, conclusions were drawn based on the results of the data analysis.

RESEARCH RESULTS AND DISCUSSION

In the preparing for the experiment phase, the researcher designed a series of learning activities to be carried out by the teacher and students over two meetings. These learning activities were intended to support the teacher in better preparing students for the learning process. The activities are as follows:

Table 1. Learning Activities in the HLT for Cube and Cuboid Topics

Session	Activities	Tujuan
1	Observing cube and cuboid frameworks	Students are able to identify the elements of cubes and cuboids (faces, edges, vertices, face diagonals, space diagonals, and diagonal planes).
	Arranging post-it papers (cube nets)	- Students are able to discover and draw various cube nets. Students are able to recognize patterns in cube nets.
	Identifying the uniqueness of dice	- Students are able to fill in the dots on cube nets following the dot-sum rules of dice.
	Arranging post-it papers (cuboid nets)	Students are able to discover and draw various cuboid nets.
2	Reviewing cube and cuboid nets	Students are able to rediscover the formulas for the surface area of cubes and cuboids.
	Determining the surface area of unit cube structures	Students are able to solve problems related to the surface area of cubes and cuboids.

HLT Cycle 1 Trial

The HLT designed during the preparing for the experiment phase was trialed with six students representing high, medium, and low ability levels. The activities conducted during the first session included observing cube and cuboid frameworks, arranging post-it papers, and discovering the uniqueness of dice. At the beginning of the activity, students were asked to mention examples of cubes found in everyday life. After observing cube and cuboid frameworks—where raffia strings were tied to the face diagonals and space diagonals, and orange-colored cardboard was placed on the diagonal planes—students were able to identify and point out the faces, edges, vertices, face diagonals, space diagonals, and diagonal planes. Using six square-shaped post-it papers, each group was asked to discover various cube nets and assign a representative to stick them on the board. The teacher then guided students to classify the cube nets based on their similarities. Students grouped cube nets shaped like the letters 'T' and 'Z' into the same category, recognizing a common structure: one square on the left, four in the middle, and one on the right. Based on this reasoning, the teacher led the students to identify cube net patterns such as 1-4-1, 1-3-2, 2-2-2, and 3-3. Each group was then given a die and asked to discover its unique characteristic—that the number of dots on opposite sides always adds up to seven. Finally, students were asked to choose four cube nets, each representing one of the identified patterns, and to fill in the dots on the cube nets according to the rule of opposite sides summing to seven.

The second session consisted of three activities: arranging post-it papers, calculating the surface area of cube and cuboid nets, and determining the surface area of a structure made from unit cubes. Using six rectangular post-it papers (three pairs of different sizes) and cube net patterns, each group was asked to find as many cuboid nets as possible. However, each group only managed to find cuboid nets with the 1-4-1 pattern. Students were then asked to work in groups to select one cube net and one cuboid net, and to draw them in the designated boxes. Next, they labeled the edges on the cuboid net using length (l), width (w), and height (h), and labeled the cube net edges with side length (s). Students were then instructed to calculate the surface area of the selected nets. Afterward, each group was asked to build a unique structure using the provided unit cubes. Once completed, they were tasked with determining the surface area of the structure excluding the bottom (floor) surface.

HLT Cycle 2 Trial

Based on the results of the retrospective analysis in HLT Cycle 1, the instructional materials underwent several revisions to improve their quality for use in Cycle 2. These revised materials were revalidated by experts to ensure alignment between the learning activities and the intended learning objectives. After the revisions were made, the materials were ready to be used in the second cycle. In the first session of Cycle 2, Activity 1 began with the teacher showing cube and cuboid frameworks while asking students about the elements of these shapes. The teacher then displayed a cuboid framework tied with raffia strings across its planes. It was observed that students actively expressed their understanding of the elements of cubes and cuboids using their own words, based on what they observed. In Activity 2, each group was asked to discover as many cube nets as possible using six equally sized square post-it papers. After identifying the cube nets, students were instructed to draw them on the provided grid paper.

Figure 1. Students Engaged in Exploring Cube Nets

Several cube nets were successfully discovered by the students. Group 1 found 5 cube nets (all with the 1-4-1 pattern); Group 2 found 7 cube nets (including 4 with the 1-4-1 pattern, 1 with the 1-3-2 pattern, 1 with the 2-2-2 pattern, and 1 with the 3-3 pattern); Group 3 discovered 6 cube nets (all with the 1-4-1 pattern); Group 4 found 7 cube nets (5 with the 1-4-1 pattern, 1 with the 1-3-2 pattern, and 1 with the 3-3 pattern); and Group 5 identified 6 cube nets (4 with the 1-4-1 pattern, 1 with the 1-3-2 pattern, and 1 with the 3-3 pattern). The teacher then asked each group's representative to attach the cube nets, which had been drawn on grid paper, to the whiteboard.

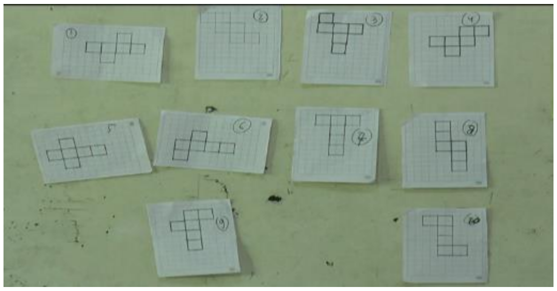

Figure 2. Cube Nets Displayed by Students on the Board

The teacher rotated net number 9 to the left and right to determine whether nets number 6 and 9 were the same. However, the teacher was still unsure, as the shapes did not appear to be exactly the same after rotation. The teacher then asked the students to prove whether nets 6 and 9 were indeed identical. Student DN bravely stepped forward to demonstrate to the class that nets 6 and 9 were the same. Instead of rotating the net left or right, DN flipped the paper over to show that the two nets were identical.

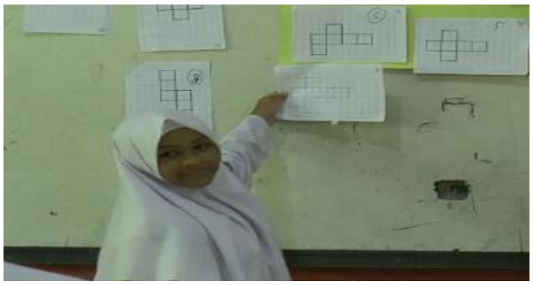

Figure 3. Student DN Proving the Equivalence of Cube Nets 6 and 9

This was an unexpected moment for the teacher, who typically only rotated the paper to the left or right. The teacher had not considered the possibility of flipping the paper over, as demonstrated by student RA. This clearly shows the development of students' spatial abilities; the student was even able to perform something neither the teacher nor the researcher had anticipated—flipping the paper over to prove that the two cube nets were indeed the same. The teacher then asked whether any of the cube nets shared similar characteristics. Student FR responded, 'They all have six square faces,' but this was not the expected answer. The teacher then selected one of the cube nets and attached it to a yellow sheet of cardboard (Figure 6), asking if there were any other cube nets with the same characteristics.Students MK, FR, and MA came to the front and attached the cube nets they believed were similar to the one already displayed. As a result, four cube nets were placed on the yellow cardboard, representing the nets the students considered to have similar properties.

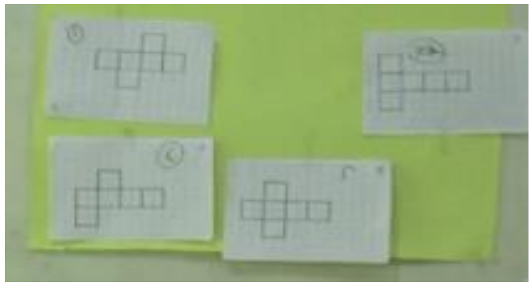

Figure 4. Cube Nets Sharing Common Features

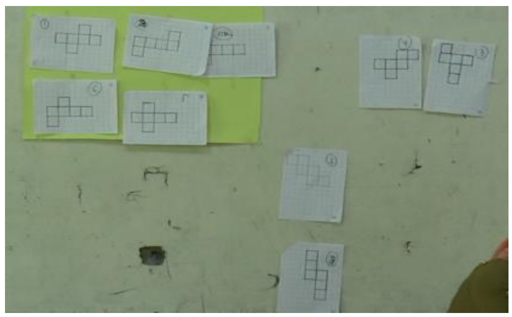

Figure 5. Categorization of Cube Nets

All four cube net patterns had been discovered, but students had only identified 9 cube nets so far. The teacher then informed the students that there are 11 cube nets in total and asked them to find the remaining two. To assist the students, the teacher briefly displayed the 'Cube Nets Guessing Game' to stimulate their thinking and asked them to observe which nets had not yet been found. The game was then closed, and students were asked to draw the two missing cube nets.

The students enthusiastically attempted to discover the remaining cube nets, even continuing to use post-it notes to physically test their findings. Some students came forward to post their cube nets on the board; however, after closer observation by their peers, it was revealed that the nets were duplicates of those already posted when flipped or rotated. Eventually, student FR successfully identified one additional net with the 1-4-1 pattern, and student RA discovered another net with the 1-3-2 pattern.

As a result, students were able to independently conclude that there are 11 cube nets in total, which can be categorized into four patterns: six nets with the 1-4-1 pattern, three with the 1-3-2 pattern, one with the 3-3 pattern, and one with the 2-2-2 pattern.

In Activity 3, the teacher distributed worksheets (LKPD) and dice to each group while asking what was unique about the arrangement of the dots on a die. The students had not yet figured it out, so the teacher gave them a clue by asking about the total number of dots on opposite faces. Student AK answered 'seven.' After discovering this property—that the number of dots on opposite faces of a die always equals seven—the students were asked to complete the worksheet (LKPD 1B).

When the allotted time ended, all groups were asked to submit their work, and a representative from each group was invited to present their results. Of the five groups, only Groups 1 and 3 completed the task correctly and thoroughly. Groups 2 and 5 only filled in dots on three cube nets, while Group 4 completed only two. Upon further examination, the researcher found that Groups 2, 4, and 5 had redrawn all four cube nets instead of directly labeling the existing ones with the dot values. This additional task took up their time, resulting in an incomplete submission.

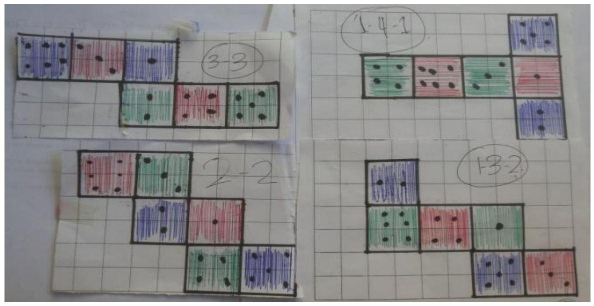

Figure 6. A Group's Attempt at Solving the Task

The conjecture for the first meeting was confirmed: students were able to identify the faces, edges, vertices, face diagonals, space diagonals, and diagonal planes of the cube and cuboid. However, they still had difficulty explaining the definitions of each of these elements. Furthermore, when the teacher asked students to attach different cube nets to the board, some students posted nets that were actually identical to ones already on the board, differing only in orientation.

The activities in the second session did not undergo any revisions. However, in the pilot experiment phase, although three activities were initially planned, some were adjusted after discussion with the research team and the model teacher. This was due to the fact that the dimensions of cuboid net faces are not uniform, unlike those of cube nets. Although students were already familiar with the 1-3-2, 2-2-2, and 3-3 patterns, they were unable to construct cuboid nets using those patterns with the post-it notes provided, as the notes came in three different sizes.

As a result, students began to assume that there were no cuboid nets other than the 1-4-1 pattern, and they also believed that cuboids had fewer possible nets compared to cubes. To address this misconception, the teacher showed the students a collection of 57 different cuboid nets.

Figure 7. Variations of Cuboid Nets

All students were surprised to learn that there were so many cuboid nets, and only then did they realize that the patterns of cuboid nets are the same as those of cube nets.

Each group was asked to choose one cuboid net, draw it on Worksheet 2 Activity 2 (LKPD 2 Kegiatan 2), and label the corresponding edges with length (l), width (w), and height (h).

Using the selected cuboid nets, students were then asked to explain how to determine the surface area of a cuboid. Based on the teacher's observation, all groups made mistakes in labeling the edges of the cuboid nets with l, w, and h. As a result, the teacher explained the concept in front of the class.

Students then discussed within their groups how to assign l, w, and h to the edges of the cuboid nets. However, many struggled to identify the correct edges, especially those who had chosen nets other than the T-shape or cross-shaped configurations. Among the five groups, only two were able to correctly assign l, w, and h. These two groups had selected T-shaped nets, which made it easier to determine the edges. The remaining three groups had selected 1-4-1 pattern nets that were neither T-shaped nor cross-shaped, which made it more difficult for them to determine the appropriate edge assignments.

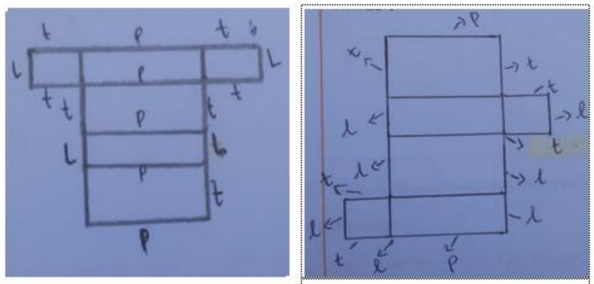

Figure 8. Student Responses in Labeling l, w, and h on Cuboid Edges

Although three student groups were unable to correctly assign length (l), width (w), and height (h), all groups were still able to derive the formula for the surface area of a cuboid. Based on interviews conducted by the researcher with those three groups, it was concluded that they visualized a complete cuboid to determine its surface area, as they found it easier to observe a 3D object than to analyze its net.

After discovering the surface area formula of a cuboid using its net, students were asked to find the surface area formula for a cube in the same way, using cube nets. All groups completed this task correctly.

The teacher then asked group representatives to present their method for deriving the formulas. Group 1 volunteered to present the method for finding the surface area of a cuboid, while Group 3 presented the method for finding the surface area of a cube.

Before students began working on routine and non-routine problems from Worksheet 2A (LKPD 2A), the teacher spontaneously gave them an additional test problem that was not included in the lesson plan (RPP 2). In an interview, the teacher explained that the purpose of this was to assess students' understanding of basic surface area problems, to determine whether they had truly grasped how to apply the formula. The teacher noted that if students could not solve routine problems, they would likely struggle even more with non-routine ones.

During the Application (A) component, after submitting their answers to the test, students were instructed to work collaboratively on the non-routine problems and submit their group work.

The conjecture in the second session was confirmed: when attempting to construct cuboid nets using post-it notes, students only discovered nets following the 1-4-1 pattern. No students were able to find nets with the 2-3-1, 2-2-2, or 3-3 patterns.

DISCUSSION

The design of the Hypothetical Learning Trajectory (HLT) based on the ELPSA framework for eighth-grade students at SMP Negeri 1 Banda Aceh was structured to encourage students to actively construct their own understanding of cube and cuboid concepts. During the learning process, the teacher used concrete teaching aids to provide students with visual experiences in recognizing mathematical concepts through representations. This aligns with the findings of John, Lalan, and Prahmana (2015), who emphasized that with the use of visual aids, students are able to construct cube and cuboid structures based on geometric ideas presented.

Additionally, the teacher asked students to explain terms and concepts related to cubes and cuboids using their own words. During this phase, students actively defined these mathematical terms and expressed their opinions about cube and cuboid-related problems. This reflects Bruner's main objective of education, which states that teachers should guide students so that they can build their own knowledge base (Suyono & Hariyanto, 2014).

Students were also asked to discuss the problems or questions provided in the worksheets (LKPD) within their groups. It was evident that students made significant efforts to understand the material through collaborative discussions. This supports the view of Suyono and Hariyanto (2014), who argued that students can more easily grasp complex concepts by discussing them in small peer groups.

Student engagement in the learning activities showed that the intended learning activities were achieved. This is consistent with the ELPSA framework, which encourages students to construct their own knowledge through interactions with their environment, including teachers and peers.

Based on the trial phase, several key findings emerged throughout the learning process. In the first session, the activity of arranging post-it notes to discover different cube nets effectively enhanced students' spatial visualization skills. This made it easier for students to understand the concept of cube nets, discover various nets, and identify net patterns.

In the second session, students showed development in their spatial visualization skills, particularly in identifying the edges of the cuboid nets as length (l), width (w), and height (h). Most students could visualize the placement of these dimensions. Through the activity of measuring the surface areas of cube and cuboid nets, students were able to rediscover the formulas for surface area. However, a few students struggled to correctly assign l, w, and h to the cuboid nets, especially those who had chosen nets from the 3-3 and 2-2-2 patterns. These patterns made it more difficult for them to determine the base and top faces, resulting in confusion when identifying the edges.

Overall, the activities conducted in both sessions were effective in enhancing students' spatial abilities. The structure of the activities required students to construct their own knowledge, aligning with Gardner (1983), who stated that spatial ability can be developed by providing children with opportunities to solve problems in their own way—either through conventional or modern methods.

The final test was administered to assess the extent to which the revised HLT (HLT 2) influenced students' spatial development regarding cube and cuboid material. The first problem addressed students' ability to identify the composition of an object whose parts had been changed or shifted. Students were asked to draw two cube nets with different patterns and fill in the dots on each net according to standard die-dot rules. Out of 32 students, 22 answered correctly; 7 made errors in placing the dots on one of the nets, and 3 drew the nets but did not fill in any dots. These results indicate that most students were able to correctly draw cube nets from different patterns, although a few struggled with determining opposing faces—particularly for nets with 2-2-2 and 3-3 patterns.

CONCLUSION

Based on the results obtained in this study, it can be concluded that the Hypothetical Learning Trajectory (HLT) designed to develop spatial ability through the ELPSA framework consists of seven core activities. The first activity is observing the structure of cubes and cuboids. The second is arranging post-it notes. The third activity involves discovering the uniqueness of dice. The fourth is calculating the surface area of cube and cuboid nets. The fifth activity compares upright and horizontal card boxes. The sixth is visualizing front and side views. The seventh activity involves identifying patterns in cube nets. These seven activities have been shown to effectively develop students' spatial abilities in learning about cubes and cuboids.

Based on these conclusions, several recommendations can be made regarding the implementation of instructional design in the classroom. Teachers intending to introduce new mathematical concepts are encouraged to begin by designing an HLT that takes into account context and inter-topic connections. This helps lay a strong foundation, making it easier for students to understand the material

.

REFERENCE

Confrey, J., Gianopulos, G., McGowan, W., Shah, M., & Belcher, M. (2017). Scaffolding learner centered curricular coherence using learning maps and diagnostic assessments designed around mathematics learning trajectories. *ZDM Mathematics Education*. Springer. Doi: 10.1007/s11858-017-0869-1.

Dalziel, J. (2015). *The Art & Science of Learning Design.* In: Maina, M., Craft, B., and Mor, Y. (Eds.). Reflections on the art and science of learning design and the larnaca declaration, pp. 3-14. AWRotterdam: Sense Publishers.

Fitri, A., & Prahmana, R. C. I. (2020). Hypothetical Learning Trajectory dalam Proses Pembelajaran Matematika.

Gardner, Howard. 1983. *Frames of Mind: The Theory of Multiple Intelligences*. New York: basic Books.

Gee, E. (2019). Kemampuan Pemecahan Masalah Matematika Melalui Alur Belajar Berbasis Realistic Mathematics Education (RME). *Jurnal Education and Development*, 7(3), 269–277.

Gravemeijer, K. (2004). Local Instruction Theories as Means of Support for Teachers in Reform Mathematics Education. Thinking and Learning, 105-128.

Gravemeijer, K., & Cobb, P. 2006. Design Research from a Learning Design Perspective. In J. Akker, K.

Lalan, Ingri Y. M., Rully Charitas Indra Prahmana, dan Peter John R. P. (2015). *Penggunaan Alat Peraga Polydron Frameworks Pada Materi Geometri Untuk Meningkatkan Kemampuan Spasial Matematis Siswa SMP Kelas VIII*.

Masithoh., Myran, F, D., Harahap, T, H., & Panggabean, E, M. (2025). Desain Lintasan Belajar dengan Model Pembelajaran Problem Based Learning untuk menemukan Rumus Luas Permukaan Tabung di Kelas IX SMP. QOSIM: Jurnal Pendidikan, Sosial & Humaniora, 3(1), 235-245

Muhsi, A., Asmika, H. & Panggabean, E, M. (2025). Desan Pembelajaran Intergral Fungsi Trigonometri Berbasis Local Instruction Theory (LIT). QOSIM: Jurnal Pendidikan, Sosial & Humaniora, 3(1), 277-283

Rahmi, (2017). Peningkatan Hasil Belajar Siswa Pada Materi Jaring-jaring Balok dan Kubus Dengan Pendekatan Contextual Teaching and Learning (CTL) Siswa Kelas IV SDN 05 Air Tawar. Jurnal Basic Edu, Universitas Muhammadiyah Semarang.

Suyono, & Hariyanto. (2014). Belajar dan Pembelajaran: Teori dan Konsep Dasar. Bandung: Remaja Rosdakarya.

Yanti, N., & Fauzan, F. (2021). Pengaruh Penggunaan Media Pembelajaran Quizizz terhadap Hasil Belajar Siswa pada Mata Pelajaran Matematika Melalui daring di Sekolah Dasar. Jurnal Basicedu, 5(5), 3109-3116. https://doi.org/10.31004/basicedu.v5i5.1288